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# The research on end-effector position and orientation error distribution of SCARA industrial robot

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***ABSTRACT :****SCARA(Selective Compliance Assembly Robot Arm) industrial robot's five parameters position and orientation error model is built by the matrix method. End-effector position and orientation error vector are derived from the combination of orthogonal testing method and the error model. A quantity of working points are selected evenly from the working space of SCARA robot. Every working point corresponds to a end-effector position and orientation error vector. Analysis on principal component is made on error vector, and the number of factors contributing to the end-effector position and orientation error of SCARA robot is reduced, thus three new factors are obtained. The position error from x axis and from y or z axis are pairwise uncorrelated. On the contrary, the position error from z axis and orientation error from x y or z axis are pairwise correlated. At last, end-effector position and orientation error is depicted by a score from the principal component analysis. Compared with the inside of working space, position and orientation error from the outside is smaller based on several error distribution sections. So the robot should avoid working in the marginal area of the outside.*
***Keywords** -SCARA industrial robot, five parameters position and orientation error model, orthogonal testing method, principal component analysis, error distribution*

## I. INTRODUCTION

SCARA robot, as depicted in Fig.1, is a kind of plane multi-joint robot containing three rotary joints and one prismatic joint. The first joint is a rotary joint, which drives upper arm to rotate; The second joint is also a rotary joint, which drives forearm to rotate. The third joint is a rotary joint, which drives tip rotary shaft to rotate. The fourth joint is a prismatic joint, which drives the tip rotary shaft to move vertically.

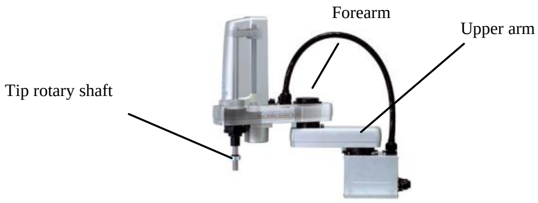


Fig.1 Selective Compliance Assembly Robot Arm

End-effector position and orientation error of SCARA robot can be depicted by position and orientation error from x, y, z axis of end-effector coordinate frame[1]. The commonly used methods to model end-effector position and orientation error of robot are matrix method and vector method. Chen et al.[2] introduced jacobian matrix to describe the relations between link parameters and end effector position and orientation error. Huang[3] introduced vector method to analyse the relations between link parameters and end effector position and orientation error.

The above mentioned two methods are modelled by four parameters model. But four parameters model would generate bad mapping when robot link axises parallel. So Hayati et al.[4]introduced a new parameter to the four parameters model, and construct a five parameters model.

Burisch[5] et al. did the research about end-effector error distribution of SCARA robot in its working area. But they only analyse one section of error distribution, which can not reflect end-effector error distribuion of three dimensional working space.

A large number of work is necessary when it comes to the analysis of end-effector error distribuion in three dimensional working space. So, Wang et al.[6] get the mechanism end-effector position and orientation error by orthogonal testing method. Only a little work is needed to get mechanism end-effector error when this method is taken.

This paper takes SCARA industrial robot for example. Firstly, five parameters position and orientation error model is built by the matrix method. Secondly, end-effector position and orientation error vector are derived from the combination of orthogonal testing method and the error model. A quantity of working points are selected evenly from the working space of SCARA robot. Every working point corresponds to a end-effector position and orientation error vector. Analysis on principal component is made on error vector, and the number of factors contributing to the end-effector position and orientation error of SCARA robot is reduced, thus some new factors are obtained. End effector position and orientation error is depicted by a score from the principal component analysis. At last, several sections of error distribution are drawn based on the score. Analysis to robot end-effector error distribuion in three dimensional working space can be made based on the sections.

## II. FIVE PARAMETERS POSITION AND ORIENTATION ERROR MODEL

2.1 SCARA robot link parameters

SCARA robot link parameters are shown in Tab.1.

Tab.1 SCARA robot link parameters

| link $i$ | $\alpha_{i-1}/(°)$ | $a_{i-1}/\text{mm}$ | $\theta_i/(°)$ | $d_i/\text{mm}$ | $\beta_{i-1}/(°)$ | range of link variables | link parameters/mm |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 90 | 0 | 0 | -140°~140° | $a_1$=250 |
| 2 | 0 | $a_1$ | 0 | 0 | 0 | -140°~140° | $a_2$=150 |
| 3 | 0 | $a_2$ | 0 | 0 | 0 | -360°~360° | |
| 4 | 0 | 0 | 0 | $d_4$ | 0 | -90~0 mm | |

2.2 Five parameters position and orientation error model

Parameters of four parameters model are . Among these parameters, depicts rotation angle of joint axis and around , depicts offset of joint axis and along , depicts rotation angle of joint axis and around , depicts offset of joint axis and along .

The adjacent joint axises of SCARA robot parallel each other, so the model of position error between adjacent joint axises is incompatible with small error model[7-8]. That is the small position error of end-effector can not be modeled by four parameters model. So Hayati et al. introduced a new parameter to the four parameters model, establishing the five parameters model[9].

In the five parameters model, jacobian matrix reflects the linear relation between link parameters space and operating space. That is, the linear relation between link parameters error and end-effector position and orientation error:

$$\boldsymbol{b} = \boldsymbol{J}\boldsymbol{X} \tag{1}$$

$\boldsymbol{J}$ isjacobian matrix; $\boldsymbol{b}$ depicts robot end-effector position and orientation error, $d_x, d_y, d_z$ depicts position error along x, y, z axis, $\delta_x, \delta_y, \delta_z$ depicts orientation error around x, y, z axis, $\boldsymbol{b} = \begin{bmatrix} d_x & d_y & d_z & \delta_x & \delta_y & \delta_z \end{bmatrix}^{\mathrm{T}}$; $\boldsymbol{X}$ depicts links parameters error.

In the five parameters model, all the link parameters are $\alpha$ $a$ $\theta$ $d$ $\beta$. The five parameters have corresponding linear relation with end-effector position and orientation error respectively[8]. So the link five parameters correspond to five jacobian matrix[8,10] as to the end-effector error: $\boldsymbol{J}_\alpha$, $\boldsymbol{J}_a$, $\boldsymbol{J}_\theta$, $\boldsymbol{J}_d$, $\boldsymbol{J}_\beta$. Jacobian matrix $\boldsymbol{J}$ can be depicted by the five matrix[11]:

$$\boldsymbol{J} = [\boldsymbol{J}_\alpha \quad \boldsymbol{J}_a \quad \boldsymbol{J}_\theta \quad \boldsymbol{J}_d \quad \boldsymbol{J}_\beta] \tag{2}$$

$\boldsymbol{J}_\alpha$ depicts the relation between $\alpha$ and end-effector position and orientation error; $\boldsymbol{J}_a$ depicts the relation between $a$ and end-effector position and orientation error; $\boldsymbol{J}_\theta$ depicts the relation between $\theta$ and end-effector position and orientation error; $\boldsymbol{J}_d$ depicts the relation between $d$ and end-effector position and orientation error; $\boldsymbol{J}_\beta$ depicts therelation between $\beta$ and end-effector position and orientation error.

## III. OBTAIN END-EFFECTOR POSITION AND ORIENTATION ERROR FROM ORTHOGONAL TESTING METHOD

Many methods are available to compute dimensional tolerances of mechanism, Caro et al.[11] provided a robust design method to synthesize its dimensional tolerances. Wang et al.[7] use Latin hypercube sampling (LHS) to compute dimensional tolerances of mechanism. This paper use orthogonal testing method to select sample of balanced matching levels of every factor. End-effector position and orientation errorvectorare derived from the combination of orthogonal testing method and the five parameters model. Then, take component extremum of every obtained position and orientation error vector asthe final position and orientation error vector.

SCARA robot has four links, and every link contains five parameters. So the robot contains twenty parameters in total. Value of parameter $\alpha$ $\beta$ $a$ of the first and fourth link ,by definition, equal zero.

That is, $\alpha_0 = 0 \quad \beta_0 = 0 \quad a_0 = 0 \quad \alpha_3 = 0$ .So fourteen available parameters are picked:

$\alpha_1, \alpha_2, a_1, a_2, \theta_1, \theta_2, \theta_3, \theta_4, d_1, d_2, d_3, d_4, \beta_1, \beta_2$ ,

and $\boldsymbol{X} = [\alpha_1 \quad \alpha_2 \quad a_1 \quad a_2 \quad \theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4 \quad d_1 \quad d_2 \quad d_3 \quad d_4 \quad \beta_1 \quad \beta_2]^{\mathrm{T}}$. If every parameter contains 2 levels, 14 parameters would correspond to 16384($2^{14}$=16384) experiments[12]. According to eq.(1), 16384 position and orientation error vectors $\boldsymbol{b}$ correspond to the experiments. It is a significant amount of work to get component extremum $d_{\mathrm{x}} \quad d_{\mathrm{y}} \quad d_{\mathrm{z}} \quad \delta_{\mathrm{x}} \quad \delta_{\mathrm{y}} \quad \delta_{\mathrm{z}}$ of 16384 obtained position and orientation errorvectors $\boldsymbol{b}$ , so orthogonal testing method is introduced to solve this problem.

The orthogonal test design table of 14 parameters, 2 levels is $\mathrm{L}_{16}(2^{14})$ [12]. Every column of the table correspond to a parameter, and every row of the table correspond to one experiment of balanced matching levels of every factor, as displayed in Tab.2. In the process of design experiment, 2 levels are selected for every parameter. For length parameters, class IT5 is set as level 2, half of class IT5 is set as level 1. For angle parameters, micron order is set as level 2, half of level 2 is set as level 1[6,13], as displayed in Tab.3.

Tab.2 Orthogonal experiment table of $\mathrm{L}_{16}(2^{14})$

| factor | $\alpha_1$ | $\alpha_2$ | $a_1$ | $a_2$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $\beta_1$ | $\beta_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 4 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| 5 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 |
| 6 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 1 |
| 7 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 1 |
| 8 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 1 | 2 |
| 9 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 |
| 10 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 2 |
| 11 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | 1 | 2 | 2 | 1 | 2 |
| 12 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 1 | 2 | 1 |
| 13 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 |
| 14 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 1 |
| 15 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | 1 |
| 16 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 1 | 2 | 2 |

Tab.3 error factors level

| | $\Delta\alpha_1$(') | $\Delta\alpha_2$(') | $\Delta a_1$/μm | $\Delta a_2$/μm | $\Delta\theta_1$(') | $\Delta\theta_2$(') | $\Delta\theta_3$(') | $\Delta\theta_4$(') | $\Delta d_1$/μm | $\Delta d_2$/μm | $\Delta d_3$/μm | $\Delta d_4$/μm | $\Delta\beta_1$(') | $\Delta\beta_2$(') |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| level 1 | 10 | 10 | 10 | 9 | 30 | 30 | 20 | 20 | 3 | 3 | 3 | 5.5 | 10 | 10 |
| level 2 | 20 | 20 | 20 | 18 | 60 | 60 | 40 | 40 | 6 | 6 | 6 | 11 | 20 | 20 |

As displayed in Tab.2, only 16 experiments is needed relative to the original 16384 experiments. So, 6 component extremums are selected from 16 obtained errorvector $\boldsymbol{b}$ , which are set asthe final position and orientation error vector component extremums.

## IV. THE DETERMINATION OF PRINCIPAL COMPONENTS AND SCORE WORKING POINTS

Principal component analysis[14] is used to transform pairwise correlated variables into pairwise uncorrelated variables, whose main purpose is to use less variables to explain most variation. So, this method can be applied to analyse the relation of 6 components of robot end-effector position and orientation error vector, and turn the 6 components into a few uncorrelated new variables[15]. End effector position and orientation error can be depicted by a score from the principal component analysis according to the obtained new variables, as described below.

4.1 Standardizing position and orientation error

Supposing the number of variables of the principal component analysis is $m$ . This variables are $u_1 \quad u_2 \quad \cdots \quad u_m$ . $n$ samples are selected, the index of variable $k$ from sample $j$ is $c_{jk}$ . The standardized index is $\tilde{c}_{jk}$ , as depicted below.

$$\tilde{c}_{jk} = \frac{c_{jk} - \mu_k}{s_k}, j = 1, 2, \cdots, n; \quad k = 1, 2, \cdots, m \tag{3}$$

$$\mu_k = \frac{1}{n} \sum_{i=1}^{n} c_{jk} \tag{4}$$

$$s_k = \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} (a_{jk} - \mu_k)^2}, \; j = 1, 2, \cdots, m \tag{5}$$

$\mu_k$ , $s_k$ are sample average and sample variance of index k respectively.

4.2 Calculate correlation coefficient matrix $\boldsymbol{R}$

correlation coefficient matrix $\boldsymbol{R} = (r_{jk})_{m \times m}$ ,

$$r_{jk} = \frac{\sum_{l=1}^{n} \tilde{c}_{lj} \cdot \tilde{c}_{lk}}{n-1}, j, k = 1, 2, \cdots, m \tag{6}$$

$r_{jj} = 1, r_{jk} = r_{kj}$ , $r_{jk}$ is the correlation coefficient of index $j$ and index $k$ .

4.3 Calculate eigenvalue and eigenvector

Calculate the eigenvalue $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_m \ge 0$ and corresponding eigenvector $\boldsymbol{v}_1 \quad \boldsymbol{v}_2 \quad \cdots \quad \boldsymbol{v}_m$ of correlation coefficient matrix $\boldsymbol{R}$ , and

$$\begin{gathered} \boldsymbol{v}_k = [v_{1k} \quad v_{2k} \quad \cdots \quad v_{mk}]^{\mathrm{T}} \\ w_1 = v_{11} \tilde{c}_1 + v_{21} \tilde{c}_2 + \cdots + v_{m1} \tilde{c}_m \\ w_2 = v_{12} \tilde{c}_1 + v_{22} \tilde{c}_2 + \cdots + v_{m2} \tilde{c}_m \\ \vdots \\ w_m = v_{1m} \tilde{c}_1 + v_{2m} \tilde{c}_2 + \cdots + v_{mm} \tilde{c}_m \end{gathered} \tag{7}$$

$w_1$ is the first principal component, $w_2$ is the second principal component, $\cdots$ , $w_m$ is the $n$th principal component.

4.4 Choose $p(p \le m)$ principal components and get scores

4.4.1 Calculate contribution rate and accumulating contribution rate of eigenvalue

$$\varphi_k = \frac{\lambda_k}{\sum_{l=1}^{m} \lambda_l}, k = 1, 2, \cdots, m \tag{8}$$

is the contribution rate of principal components $w_k$

$$\gamma_p = \frac{\sum_{l=1}^{p} \lambda_l}{\sum_{l=1}^{m} \lambda_l} \tag{9}$$

is the accumulating contribution rate of principal components $w_1 \quad w_2 \quad \cdots \quad w_p$ . The foregoing $p$ index variables $w_1 \quad w_2 \quad \cdots \quad w_p$ is selected as the $p$ principal components, when $\gamma_p$ is nearly equal to $1$ (generally $\gamma_p = 0.85, 0.90, 0.95$ ).

4.4.2 Get corresponding scores

$$\boldsymbol{S} = \sum_{k=1}^{p} \varphi_k w_k \tag{10}$$

$\varphi_k$ is the contribution rate of $k$th principal component. According to the scores, the end-effector position and orientation error can be evaluated.

## V. EXPERIMENTAL SIMULATION ANALYSIS

The working space of SCARA robot is fanshaped space, so the moving range of the first, second, and fourth joint should be divided equally to take a sample evenly. And every well-distributed sample correspond to a working point.

First, the moving range of the first, and second joint are divided into 20 equal parts respectively. The fourth joint is divided into 10 equal parts. Then 4000 well-distributed working points in the working space can be get by the above mentioned method.

The movement form of fourth joint is the displacement along axis z, so to divide moving range of the joint into 10 equal parts is to divide the working space into 10 cross sections. Then error distribution of every cross section can be studied respectively. The cross section z=0 mm is taken for an example to illustrate error distribution of SCARA robot. The error distribution of other cross sections can be inferred from this.

Second, five parameters position and orientation error model is built for every working point on section z=0 mm. 400 end-effector position and orientation error vector are derived from the combination of orthogonal testing method and the error model.

Third, Analysis on principal component is made on the 400 error vector, and the eigenvalue, eigenvector, and accumulating contribution rate are obtained, as displayed in Tab.4 and 5.

Tab.4 Eigenvalues and Contribution rates

| number | eigenvalues | contribution rates (%) | accumulating contribution rate (%) |
|---|---|---|---|
| 1 | 4.0000 | 66.6667 | 66.6667 |
| 2 | 1.0000 | 16.6667 | 83.3334 |
| 3 | 1.0000 | 16.6667 | 100 |
| 4 | 0 | 0 | 100 |
| 5 | 0 | 0 | 100 |
| 6 | 0 | 0 | 100 |

Tab.5 Eigenvectors

| vector | $\tilde{c}_1$ | $\tilde{c}_2$ | $\tilde{c}_3$ | $\tilde{c}_4$ | $\tilde{c}_5$ | $\tilde{c}_6$ |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | -0.5000 | -0.5000 | -0.5000 | -0.5000 |
| 2 | -1 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 1 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0.6142 | 0.0831 | 0.0831 | -0.7804 |
| 5 | 0 | 0 | 0 | -0.7071 | 0.7071 | 0 |
| 6 | 0 | 0 | 0.6106 | -0.4930 | -0.4930 | 0.3755 |

According to Tab.4, the contribution rate of first principal component is 66.6667%, the second one is 16.6667%, the third one is 16.6667%. The accumulating contribution rate of the three principal component is nearly 100%, which is obviously greater than 95%. The 3 principal components contain the 6 original error components, so it is reasonable. The 3 principal components are:

$$w_1 = 0\cdot\tilde{c}_1 + 0\cdot\tilde{c}_2 - 0.5\cdot\tilde{c}_3 - 0.5\cdot\tilde{c}_4 - 0.5\cdot\tilde{c}_5 - 0.5\cdot\tilde{c}_6$$

$$w_2 = -1\cdot\tilde{c}_1 + 0\cdot\tilde{c}_2 + 0\cdot\tilde{c}_3 + 0\cdot\tilde{c}_4 + 0\cdot\tilde{c}_5 + 0\cdot\tilde{c}_6$$

$$w_3 = 0\cdot\tilde{c}_1 + 1\cdot\tilde{c}_2 + 0\cdot\tilde{c}_3 + 0\cdot\tilde{c}_4 + 0\cdot\tilde{c}_5 + 0\cdot\tilde{c}_6$$

According to Tab.5, first two components of first principal component are 0, absolute value of the left four components are 0.5. So position error along axis z and orientation error around axis x, y, z are correlated, and linear combination of them is regarded as the first new index.

The absolute value of first component of second principal component is 1, the left five components are 0. So position error along axis x is uncorrelated with the other components. The linear combination of them is regarded as the second new index.

The absolute value of second component of third principal component is 1, the left five components are 0. So position error along axis y is uncorrelated with the other components. The linear combination of them is regarded as the third new index.

The chosen 3 principal components are pairwise uncorrelated, and contain the 6 components of end effector position and orientation vector b. 6 original index of vector b are replaced by 3 new index properly. Number of factors contributing to the end-effector position and orientation error is reduced.

Fourth, principal component analysis is made for the 400 position and orientation error samples on section z=0 mm based on 3 new index. 400 scores is obtained correspond to the 400 samples, as displayed in Tab.6.

Tab.6 Comprehensive scores

| score | 1 | 2 | 3 | 4 | 5 | ⋯ | 20 |
|---|---|---|---|---|---|---|---|
| number | 0.9310 | 0.9390 | 0.9687 | 1.0262 | 1.0998 | ⋯ | 1.6252 |
| 1 | 0.9310 | 0.9390 | 0.9687 | 1.0262 | 1.0998 | ⋯ | 1.6252 |
| 2 | 0.9310 | 0.9390 | 0.9687 | 1.0262 | 1.0998 | ⋯ | 1.6252 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋯ | ⋮ |
| 20 | 0.9310 | 0.9390 | 0.9687 | 1.0262 | 1.0998 | ⋯ | 1.6252 |

According to Tab.6, 400 scores can be divided into 20 groups depending on values, and every group contains 20 same values. Indices of position and orientation error corresponding to higher scores have greater contribution to end-effector error. On the contrary, indices of position and orientation error corresponding to lower scores have less contribution to end-effector error. So, 10 lower group scores are regarded as the range of smaller end-effector error. 10 higher group scores are regarded as the range of bigger end-effector error. Error distribution of section z=0 mm are drawn based on the 400 scores, as displayed in Fig.2. Similarly, error distribution of section z=-10mm, -20mm, -30mm, -40mm, -50mm, -60mm, -70mm, -80mm, -90mm are depicted in Fig.3, Fig.4, and Fig.5 respectively.

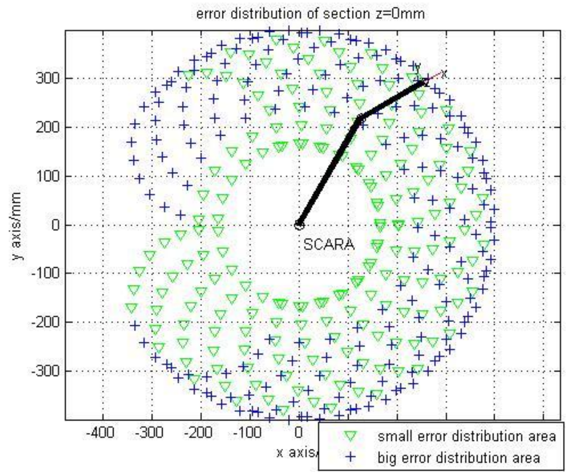


Fig.2 Position and orientation error distribution of section z=0mm

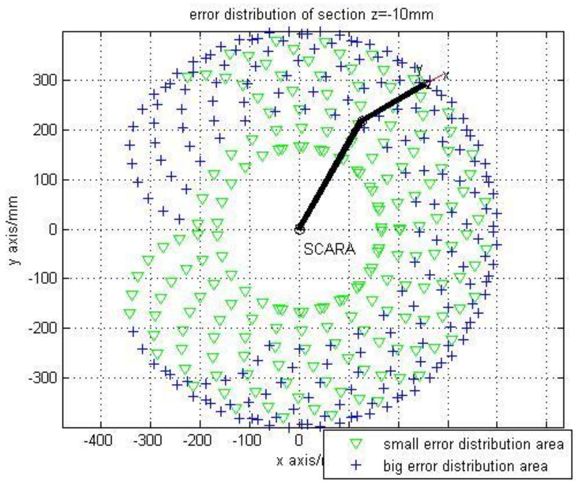


Fig.3 Position and orientation error distribution of section z=-10mm

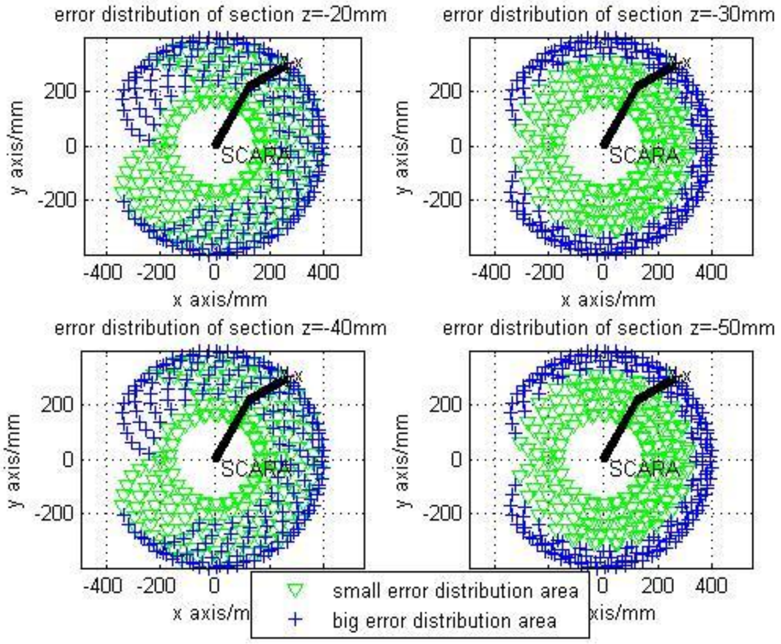


Fig.4 Position and orientation error distribution of sections z=-20mm,-30mm,-40mm,-50mm respectively

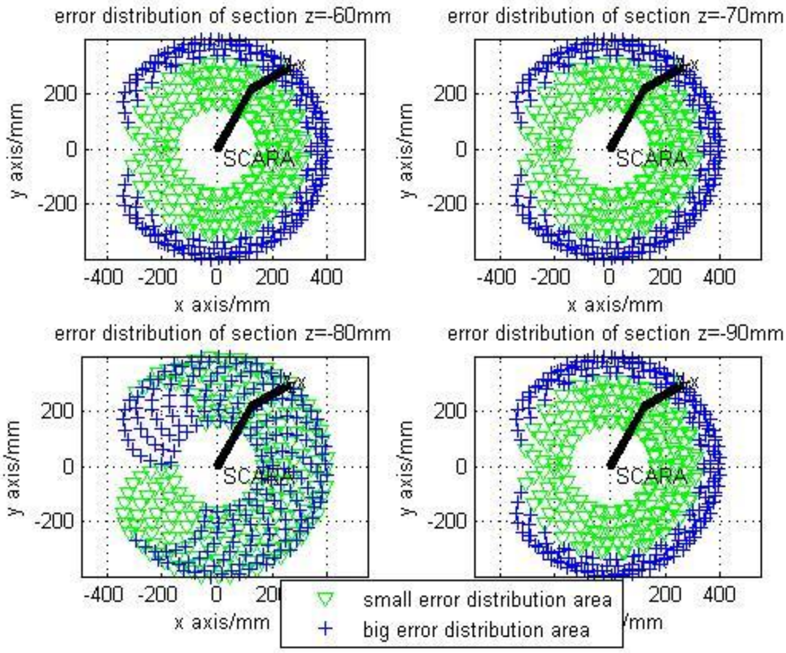


Fig.5 Position and orientation error distribution of sections z=-60mm,-70mm,-80mm,-90mm respectively

Green triangle area in Fig.2-Fig.5 is the area with smaller position and orientation error of robot, and it is an ideal area for robot to work in. Blue cross area is the area with bigger position and orientation error of robot, and it is an area the robot should avoid working in. Area where green triangle and blue cross mixed with each other is an area that error distribution is irregular. So it is also an area the robot should avoid working in.

On the sections z=0 mm, -10 mm, -20 mm, -40 mm, 30% of the inside area is distributed with smaller position and orientation error of robot, and 70% of the outside area is mixed with smaller and bigger position and orientation error. On the sections z=-30 mm, -50 mm, -60mm, -70mm, -90mm, 70% of the inside area is distributed with smaller position and orientation error, and 30% of the outside area is distributed with bigger position and orientation error. On the section z=-80mm, both the inside and the outside of the error distributed area are mixed with smaller and bigger position and orientation error.

To conclude, position and orientation error in the inside area of the working space is smaller than the outside area. So robot should avoid working in the outside marginal working space.

## VI. CONCLUSION

This paper takes SCARA industrial robot for example. Firstly, five parameters position and orientation error model is built by the matrix method. Secondly, end-effector position and orientation error vector are derived from the combination of orthogonal testing method and the error model. A quantity of working points are selected evenly from the working space of SCARA robot. Every working point corresponds to a end-effector position and orientation error vector. Analysis on principal component is made on error vector, and the number of factors contributing to the end-effector position and orientation error of SCARA robot is reduced, thus 3 new factors are obtained. End-effector position and orientation error is depicted by a score from the principal component analysis.

The position error from x axis and from y or z axis are pairwise uncorrelated. On the contrary, the position error from z axis and orientation error from x y or z axis are pairwise correlated.

At last, several sections of error distribution are drawn based on the score. Analysis to robot end-effector error distribuion in three dimensional working space can be made based on the sections. Position and orientation error in the inside area of the working space is smaller than the outside area. So robot should avoid working in the outside marginal working space. An experiment still needs to be conducted with the prototype of SCARA robot to demonstrate the menthod to depict error distribution is reasonable.

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